NAG Toolbox for MATLAB

s14ac

1 Purpose

s14ac returns a value of the function $\psi(x) - \ln x$, where ψ is the psi function $\psi(x) = \frac{d}{dx} \ln \Gamma(x) = \frac{\Gamma'(x)}{\Gamma(x)}$.

2 Syntax

```
[result, ifail] = s14ac(x)
```

3 Description

s14ac returns a value of the function $\psi(x) - \ln x$. The psi function is computed without the logarithmic term so that when x is large, sums or differences of psi functions may be computed without unnecessary loss of precision, by analytically combining the logarithmic terms. For example, the difference $d = \psi\left(x + \frac{1}{2}\right) - \psi(x)$ has an asymptotic behaviour for large x given by $d \sim \ln\left(x + \frac{1}{2}\right) - \ln x + O\left(\frac{1}{x^2}\right) \sim \ln\left(1 + \frac{1}{2x}\right) \sim \frac{1}{2x}$.

Computing d directly would amount to subtracting two large numbers which are close to $\ln\left(x + \frac{1}{2}\right)$ and $\ln x$ to produce a small number close to $\frac{1}{2x}$, resulting in a loss of significant digits. However, using this function to compute $f(x) = \psi(x) - \ln x$, we can compute $d = f\left(x + \frac{1}{2}\right) - f(x) + \ln\left(1 + \frac{1}{2x}\right)$, and the dominant logarithmic term may be computed accurately from its power series when x is large. Thus we avoid the unnecessary loss of precision.

The function is derived from the function PSIFN in Amos 1983.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Amos D E 1983 Algorithm 610: A portable FORTRAN subroutine for derivatives of the psi function *ACM Trans. Math. Software* **9** 494–502

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

Constraint: **x** > 0.0.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, $\mathbf{x} \leq 0.0$. s14ac returns the value zero.

ifail = 2

No result is computed because underflow is likely. The value of **x** is too large. s14ac returns the value zero.

ifail = 3

No result is computed because overflow is likely. The value of **x** is too small. s14ac returns the value zero.

7 Accuracy

All constants in s14ac are given to approximately 18 digits of precision. Calling the number of digits of precision in the floating-point arithmetic being used t, then clearly the maximum number of correct digits in the results obtained is limited by $p = \min(t, 18)$.

With the above proviso, results returned by this function should be accurate almost to full precision, except at points close to the zero of $\psi(x)$, $x \simeq 1.461632$, where only absolute rather than relative accuracy can be obtained.

8 Further Comments

None.

9 Example

```
x = 0.1;
[result, ifail] = s14ac(x)
```

```
result =
   -8.1212
ifail =
           0
```